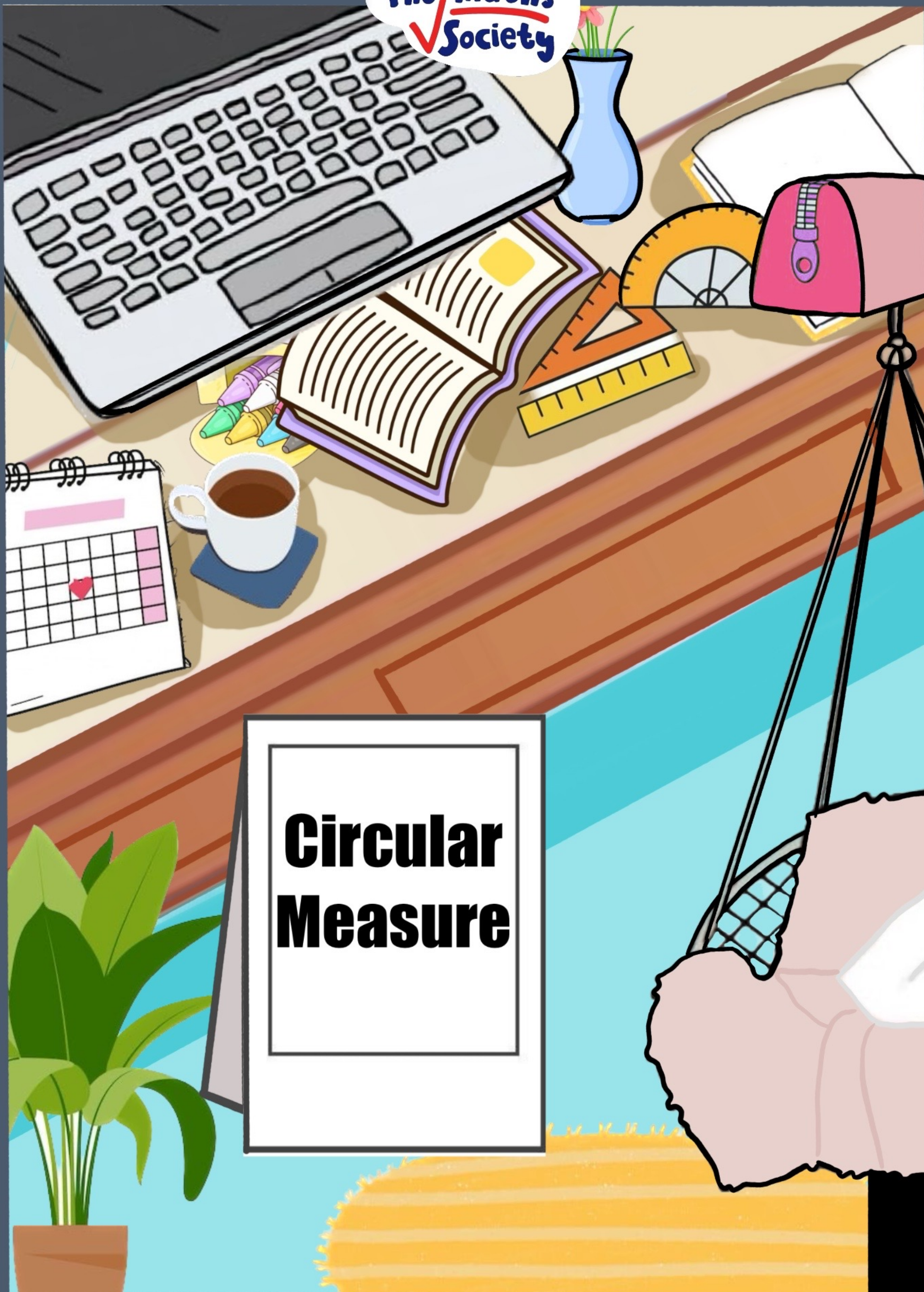
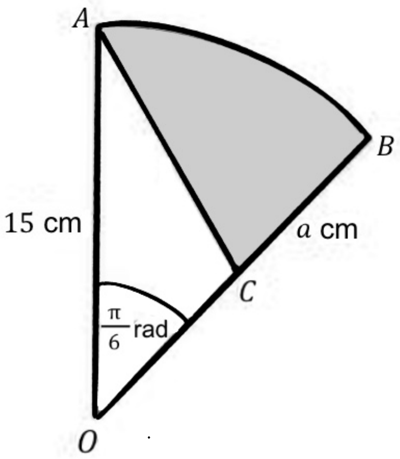
The Maths Society



Circular Measure

1.



The diagram shows the sector AOB of a circle, centre O and radius 15 cm. Angle AOB is $\frac{\pi}{6}$ radians. Point C lies on OB such that CB is a cm. AC is a straight line.

(a) Find the exact value of a such that the area of triangle AOC is equal to the area of the shaded region ACB. [4]

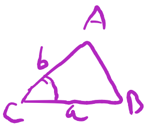
Area of AOC $= \frac{1}{2}\times 15(15-a)\sin\frac{\pi}{6} = \frac{15}{4}(15-a)$

Area of ACB $= \frac{1}{2}\times\frac{\pi}{6}\times 15^2 - \frac{1}{2}\times 15(15-a)\sin\frac{\pi}{6}$

$\frac{15^2}{12}\pi = \frac{15}{2}(15-a)$

$\frac{5}{2}\pi = 15-a$

$a = 15 - \frac{5}{2}\pi$ cm

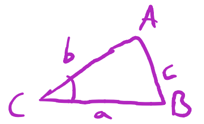


area of Δ $= \frac{1}{2}ab\sin C$

area of sector (rad):

$A = \frac{\theta r^2}{2}$

(b) For the value of a found in **part (a)**, find the perimeter of the shaded region. Give your answer correct to 1 decimal place. [3]



cosine rule:

$c^2 = a^2+b^2-2ab\cos(C)$

length of arc (rad):

$l = \theta r$

$\overrightarrow{AC} = \sqrt{15^2 + \left(\frac{5}{2}\pi\right)^2 - 2\times 15\times\frac{5}{2}\pi\times\cos\left(\frac{\pi}{6}\right)}$

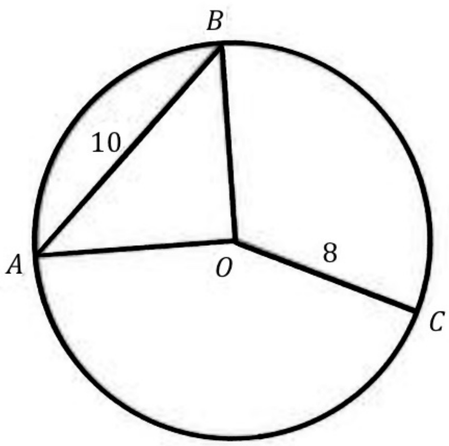
$= 9.09$ cm

$\overrightarrow{AB} = \frac{\pi}{6}\times 15$

$= \frac{5}{2}\pi$ cm

$\therefore$ Perimeter of shaded area $= 24.1$ cm

2. In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle, centre O, radius 8. The points A, B and C lie on the circumstance of the circle. The chord AB has length 10.

(a) Show the angle BOA is 1.35 correct to 2 decimal places.

[2]

$\overrightarrow{OA} = 8$ cm as radius of circle

$$(\overrightarrow{AB})^2 = (\overrightarrow{OA})^2 + (\overrightarrow{OB})^2 - 2(\overrightarrow{OA})(\overrightarrow{OB})\cos(\angle BOA)$$

$$\angle BOA = \cos^{-1}\left[\frac{(\overrightarrow{OA})^2 + (\overrightarrow{OB})^2 - (\overrightarrow{AB})^2}{2(\overrightarrow{OA})(\overrightarrow{OB})}\right] = \cos^{-1}\left(\frac{8^2 + 8^2 - 10^2}{2 \times 8 \times 8}\right)$$

$$= 1.35 \text{ rad } (2\text{dp})$$

(b) Given that the minor arc BC has a length of 18, find angle BOC.

[2]

$$l = \theta r$$

$$\theta = \frac{18}{8} = 2.25 \text{ rad}$$

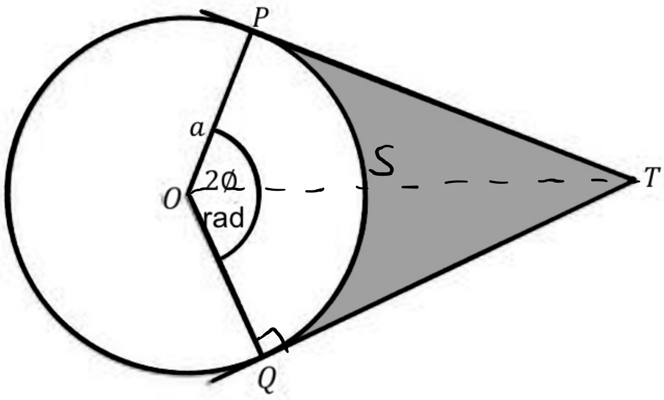
(c) Find the area of the minor sector AOC.

[3]

$$A = \frac{\theta r^2}{2} = \frac{(2\pi - 2.25 - 1.35) \times 8^2}{2}$$

$$= 85.86 \text{ cm}^2 \quad (2\text{dp})$$

3. In this question all lengths are in centimetres.



The diagram shows a circle, centre O,radius a.The lines PT and QT are tangents to the circle at P and Q respectively. Angle POQ is $2\emptyset$ radians.

(a) In the case when the area of the sector OPQ is equal to the area of the shaded region, show that $\tan\emptyset = 2\emptyset$.

[4]

area of OSQ $= \frac{\emptyset a^2}{2}$ $\quad \overrightarrow{OT} = \frac{a}{\cos\emptyset}$

area of OTQ $= \frac{1}{2} a \times \frac{a}{\cos\emptyset} \sin\emptyset$

$= \frac{1}{2} a^2 \tan\emptyset$

$2 \times \frac{\emptyset \cancel{a^2}}{\cancel{2}} = \cancel{\tfrac{1}{2}} \cancel{a^2} \tan\emptyset$ as area of OTQ = 2 × area of OSQ

$\tan\emptyset = 2\emptyset$ //

(b) In the case when the perimeter of the sector OPQ is equal to half the perimeter of the shaded region, find an expression for tan ∅ in terms of ∅. [3]

Perimeter of $OPQ = 2a + 2\phi a$

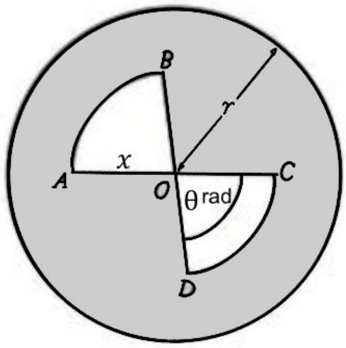
Perimeter of $PTQ = 2a\tan\phi + 2\phi a$

$$4\cancel{a} + 4\phi\cancel{a} = 2\cancel{a}\tan\phi + 2\phi\cancel{a}$$

$$4 + 2\phi = 2\tan\phi$$

$$\tan\phi = 2 + \phi$$

4.



The diagram shows a circle with centre O and radius r. OAB and OCD are sectors of a circle with centre O and radius x, where $0 < x \leq r$. Angle AOB = angle COD = θ radians, where $0 < \theta < \pi$.

(a) Find, in terms of r, x and θ, the perimeter of the shaded region. [3]

$L = \theta x \quad P = 2\pi r + 2\theta x + 4x$

(b) Find, in terms of r, x and θ, the area of the shaded region. [1]

$A = \frac{\theta x^2}{2} \quad A = \pi r^2 - 2 \times \frac{\theta x^2}{2}$

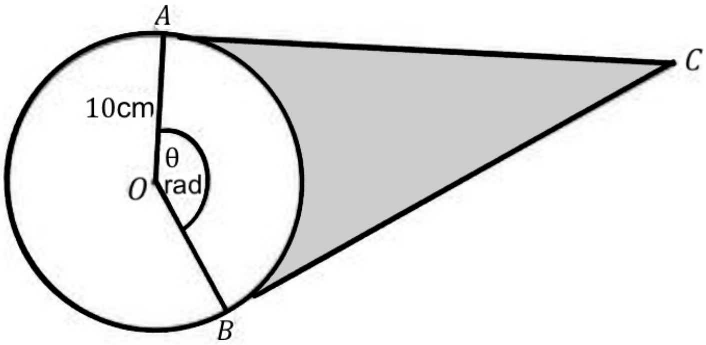
$= \pi r^2 - \theta x^2$

It is given that x can vary and that r and θ are constant.

(c) Write down the least possible area of the shaded region in terms of r and θ. [2]

$x = r \quad A = \pi r^2 - \theta r^2$

5.



The diagram shows a circle, centre O,radius 10cm.The points A and B lie on the circumference of the circle.The tangent at A and the tangent at B meet at the point C.The angle AOB is θ radians.The length of the the minor arc AB is 28cm.

(a) Find the value of θ. [1]

$C = \theta r$

$\theta = \frac{28}{10} = 2{\cdot}8$ rad

(b) Find the perimeter of the shaded region. [3]

$\overrightarrow{BC} = 10\tan 1{\cdot}4$

$P = 20\tan 14 + 28$

$= 144$ cm

(c) Find the area of the shaded region. [3]

$\angle ACB = (\pi - 2.8)$ rad

Area of $\triangle ACB = \frac{1}{2} \times [10\tan(1.4)]^2 \sin(\pi - 2.8)$
$= 563.04 \text{ cm}^2$

Area of $\triangle AOB = \frac{1}{2} \times 10^2 \sin 2.8$
$= 16.75 \text{ cm}^2$

Area of $AOB = \frac{2.8 \times 10^2}{2}$
$= 140 \text{ cm}^2$

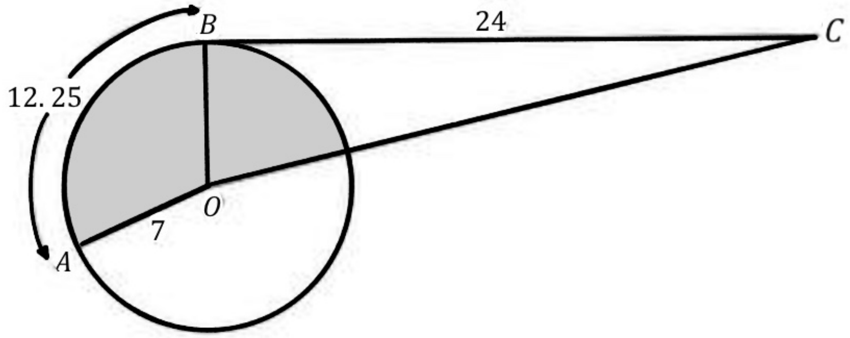
$563.04 + 16.75 - 140 = 439.79$
$= 440 \text{ cm}$

or Area of $\triangle AOC = \frac{1}{2} \times 10\tan(1.4) \times 10$
$= 289.89 \text{ cm}^2$

Area of minor sector $= \frac{2.8 \times 10^2}{2}$
$= 140 \text{ cm}^2$

shaded area $= 2 \times 289.89 - 140$
$= 439.78$
$= 440 \text{ cm}$

6. In this question all lengths are in metres.



The diagram shows a circle, centre O, radius 7.The points A and B lie on the circumference of the circle.The line BC is a tangent to the circle at the point B such that the length of BC is 24.The length of the minor arc AB is 12.25.

(a) Find the obtuse angle AOB,giving your answer in radians. [1]

$l = \theta r$

$\theta = \frac{12.25}{7} = 1.75\,\text{rad}$

(b) Find the perimeter of the shaded region. [4]

$\angle BOC = \tan^{-1}\left(\frac{24}{7}\right)$

$P = 2 \times 7 + \left(\tan^{-1}\left(\frac{24}{7}\right) + 1.75\right) \times 7$

$= 35.26\text{ m (2dp)}$

(c) Find the area of the shaded region. [2]

$$A = \frac{\left[\tan^{-1}\left(\frac{24}{7}\right)+1.75\right] \times 7^2}{2}$$

$$= 74.41\, m^2 \quad (2dp)$$